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Vectors in 2D

deffinition: a vector is a directed line segment connecting 2 points together

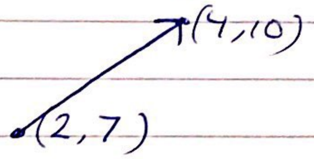
$v = \langle 2, 3 \rangle$ → must be writen with $\langle\ \rangle$

→ $= 2i + 3j$

→ by deviding j/i you get slope$\left(\frac{3}{2}\right)$

if given the vector $v = \langle 2,3 \rangle$ from a point $(2,7)$

1st: add i and j to the point to find the end point



Initial point: the point where the vector starts.

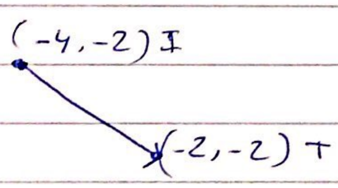
Terminal point: the end point of a vector

Note: if given IP then add i and j

if given TP then subtract i and j

Question: $v = \langle 2, -4 \rangle$ $IP = (-4, 2)$

$TP = (-2, -2)$



$$|v| = \sqrt{\Delta x^2 + \Delta y^2}$$

$$= \sqrt{2^2 + (-4)^2}$$

$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

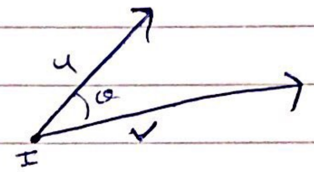
Dot product $u \cdot v$

$$u = \langle x_1 , y_1 \rangle$$
$$v = \langle x_2 , y_2 \rangle$$

$$u \cdot v = (x_1 \cdot x_2) + (y_1 \cdot y_2)$$

If 2 vectors have a dot-product of 0 then they are perpendicular.

To find the angle b/w.



$$\cos\theta = \frac{u \cdot v}{|u|\,|v|}$$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u|\,|v|}\right)$$

Question: $u = \langle 3, 2 \rangle$ $v = \langle -2, 7 \rangle$

$\cos\theta = 8/|u||v|$

$|u| = \sqrt{3^2+2^2}$

$= \sqrt{13}$

$|v| = \sqrt{4+49}$

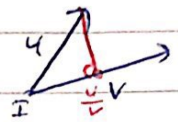
$= \sqrt{53}$

$\theta = \cos^{-1}(\text{ans})$

$= 72.26^\circ$

Projections.

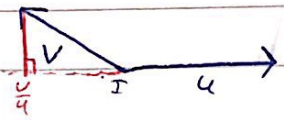
projection $\frac{u}{v}$



1st: go to the TP of u
and draw a line $\perp$ to
v.

→ The vector $[I\ \frac{u}{v}]$ is the projection.

projection $\frac{v}{u}$



1st: exetent u
2nd: go to TP of V
and draw $\perp$ to u(extention)

→ $[I\ \frac{v}{u}]$ is projection

Formulas.

$$\text{proj } \frac{v}{u} : \frac{v \cdot u}{|u|^2} \cdot u$$

$$\text{proj } \frac{u}{v} : \frac{u \cdot v}{|v|^2} \cdot v$$

$$\text{length of proj } \frac{v}{u} = \frac{|u \cdot v|}{|u|}$$

$$\text{length of proj } \frac{u}{v} = \frac{|u \cdot v|}{|v|}$$

Vectors in 3D

$v = \langle 2, 3, 1 \rangle$
$u = \langle -2, 4, -2 \rangle$

$u \cdot v = (x_1 \cdot x_2) + (y_1 y_2) + (z_1 z_2)$

Question. Find parametric equations of a line that passes through Q_1 Q_2

$Q_1 = (2, 3, -1)$
$Q_2 = (4, 0, -5)$

1st: create a vector $\overrightarrow{QQ_2}$

$$\overrightarrow{Q_1Q_2} = Q_2 - Q_1$$
$$= 4-2,\ 0-3,\ -5+1$$
$$= \langle 2, -3, -4 \rangle$$

2nd: multiply the v by t and add Q_1

$$(2, 3, -1) + t\langle 2, -3, -4 \rangle \quad t \in \mathbb{R}$$

$$L = (2+2t, 3-3t, -1-4t)$$

$$\left.\begin{aligned} x &= 2+2t \\ y &= 3-3t \\ z &= -1-4t \end{aligned}\right\} t \in \mathbb{R}$$

= parametric equations

Parallel

in 2D: 2 lines that never meet
in 3D: 2 lines that have the same directed vector and never intersect.

$$V_1 = t\,V_2$$

Question $L_1 \Rightarrow \begin{cases} x = 2+3t \\ y = -2t \\ z = 1+3t \end{cases} \quad t \in \mathbb{R}$

$L_2 \Rightarrow \begin{cases} x = 12w \\ y = -8w \\ z = 10+12w \end{cases} \quad w \in \mathbb{R}$

are they parallel ?

1st: check if they satisfy $V_1 = t V_2$

$V_1 = \langle 3, -2, 3 \rangle \qquad V_2 = \langle 12, -8, 12 \rangle$

$12 = \boxed{4}(3)$ ✓ $\qquad V_2 = 4V_1$
$-8 = \boxed{4}(-2)$
$12 = \boxed{4}(3)$

2nd: to check if they are parallel or overlapping.
choose a point and see if it intersects both lines.

$VL_1 = \langle 3, -2, 3 \rangle$ $\quad VL_2 = \langle 12, -8, 12 \rangle$

$PL_1 = (2, 0, 1)$ $\quad PL_2 = (0, 0, 10)$

1st: take 1 point and the opposite V

PL_1 and VL_2

point x = $VL_2 x$
point y = $VL y$
point z = $VL z$

$$\left.\begin{aligned} 2 &= 12w \\ 0 &= -8w \\ 1 &= 10 + 12w \end{aligned}\right\} \text{if } w = w = w$$

Perpendicular.
Dot product must $= 0$ and intersect at 1 point.

Question: $L_1 \Rightarrow \left.\begin{aligned} x &= 2 + 4t \\ y &= -4 - 2t \\ z &= 1 + t \end{aligned}\right\} t \in \mathbb{R}$

1st: Dot product $= 0$
$\langle 4, -2, 1 \rangle \; v_1$
$\langle -2, -3, 2 \rangle \; v_2$
$DP = -8 + 6 + 2$
$= 0$

$L_2 \Rightarrow \left.\begin{aligned} x &= 3 - 2w \\ y &= 3w \\ z &= -2 + 2w \end{aligned}\right\} t \in \mathbb{R}$

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Scanned with CamScanner

2nd: check for 1 intersection.

rearange formulas into $x_1 = x_2$ and $y_1 = y_2$

$$2 + 4t = 10 - 2w \qquad\qquad -4 - 2t = -3w$$
$$4t + 2w = 10 - 2 \qquad\qquad -2t + 3w = 4$$

$\rightarrow 4t + 2w = 8$

$\rightarrow -2t + 3w = 4$

Option 1. calculations on paper.

$$t = \frac{\begin{vmatrix} 8 & 2 \\ 4 & 3 \end{vmatrix}}{\begin{array}{cc} t & w \\ \multicolumn{2}{c}{\begin{vmatrix} 4 & 2 \\ -2 & 3 \end{vmatrix}} \end{array}} \qquad\qquad w = \frac{\begin{vmatrix} 4 & 8 \\ -2 & 4 \end{vmatrix}}{\begin{array}{cc} t & w \\ \multicolumn{2}{c}{\begin{vmatrix} 4 & 2 \\ -2 & 3 \end{vmatrix}} \end{array}}$$

$$\frac{\times \; - \; \times}{\times \; - \; \times}$$

then substitute t and w into z_1 and z_2

if $z_1 = z_2 \therefore \perp$

if $z_1 \neq z_2 \therefore \cancel{\perp}$

point of intersection = ~~[illegible]~~

(W_x, W_y, W_z)

or (t_x, t_y, t_z) The same.

Cross product $u \times v$

$$\begin{array}{cccc} & i & j & k \\ u = \langle & 2, & 1, & -1 \rangle \\ v = \langle & 3, & 1, & 4 \rangle \end{array}$$

1st: Draw the chart and subsitute i, j and k.

	i	j	k
u =	2	1	-1
v =	3	1	4

Formula =

$$(5)i - (11)j + (-1)k$$

vector = $\langle 5, -11, -1 \rangle$

2nd: calculate the co-oficents.

i	j	k
2	1	-1
3	1	4

$(1 \times 4) - (-1 \times 1)$
$= 4 + 1$
$= 5i$

i	j	k
2	1	-1
3	1	4

$(2 \times 4) - (-1 \times 3)$
$= 8 + 3$
$= 11j$

i	j	k
2	1	-1
3	1	4

$(2 \times 1) - (1 \times 3)$
$= 2 - 3$
$\Rightarrow -1k$

Applications of the cross product

1: to find a vector that is perpendicular $\perp$ to two other.

2: to find the area of a triangle in space.

3: To find the distence between a poin and a line in space

4: to find the equation of a plane.

1st: to find a vector (n) that is perpendicular to $\vec{u}$ and $\vec{v}$. $u = \langle 2, 1, 3 \rangle$ $v = \langle 3, 2, 4 \rangle$

$n = u \times v$

i	j	k
2	1	3
3	2	4

$i = 4 - 6$
$= -2$
$j = 8 - 9$
$= -1$
$k = 4 - 3$
$= 1$

$n = -2i + j + k$
$= \langle -2, 1, 1 \rangle$

to check: $n \cdot u$ and $n \cdot v$ must $= 0$

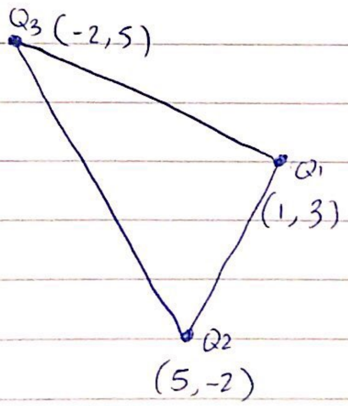
$n \cdot u = -4 + 1 + 3$
$= 0$ ✓✓

$n \cdot v = -6 + 2 + 4$
$= 0$ ✓✓

2nd: to find the area of a triangle in space, given 3 points $Q_1 = (1, 3)$ $Q_2 = (5, -2)$ $Q_3 = (-2, 5)$

1st: choose a point as the initial

$I = Q_1$



2nd: Find the ~~length~~ vectors $\overrightarrow{Q_1Q_2}$ and $\overrightarrow{Q_1Q_3}$

$u = \overrightarrow{Q_1Q_2} = \langle 4, -5, 0 \rangle$
$V = \overrightarrow{Q_1Q_3} = \langle -3, 2, 0 \rangle$

$\text{Area} = |u \times v| \frac{1}{2}$

$A = 0i + 0j - 7k$

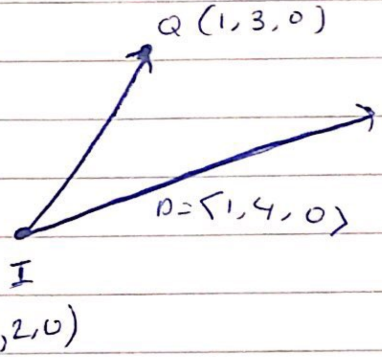
$|A| = \sqrt{7^2} \times \frac{1}{2}$
$= 3.5 \text{ units}^2$

3rd: Find the distance between a point and a line in space. $Q = (1, 3, 0)$

$L = 4x + 2$

1st: write the parametric for L

$$\left.\begin{aligned} x &= t \\ y &= 4t + 2 \\ z &= 0 \end{aligned}\right\} t \in \mathbb{R}$$



2nd: find the Directional vector and a point.

$D = \langle 1, 4, 0 \rangle$

$p = (0, 2, 0)$

Formulas

$$|QL| = \frac{|\vec{D} \times \vec{IQ}|}{|D|}$$

3rd: Find $\vec{IQ}$ and $|D|$.

$\vec{IQ} = \langle 1, 1, 0 \rangle$

$$\begin{aligned} |D| &= \sqrt{1^2 + 4^2} \\ &= \sqrt{1 + 16} \\ &= \sqrt{17} \end{aligned}$$

4th: solve

$|QL| = |\vec{D} \times \vec{IQ}| / |D|$

$$\begin{aligned} |\vec{D} \times \vec{IQ}| &= 0i + 0j - 3k \\ &= \sqrt{3^2} \\ &= 3 \end{aligned}$$

$$|QL| = \frac{3}{\sqrt{17}}$$

4th: To find the equation of a plane given 3 points.

$Q_1 = (1,1,0)$ $Q_2 = (0,2,4)$ $Q_3 (2,3,0)$

note: → if N is a vector ⊥ to a plane, then N is perpendicular to every vector on the plane.

→ Needs 3 points to determine a unique plane (must not be colinear)

1st: find N by $\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

i	j	k
-1	1	4
1	2	0

$\overrightarrow{Q_1Q_2} = \langle -1, 1, 4 \rangle$

$\overrightarrow{Q_1Q_3} = \langle 1, 2, 0 \rangle$

$N = -8i + 4j - 3k$

$\langle -8, 4, -3 \rangle$

2nd: choose any of $Q_{1,2,3}$ and supstitute it and N in the formula.

$$N_x (x - P_x) + N_y (y - P_y) + N_z (z - P_z) = 0$$

$N = \langle \underset{N_x}{-8}, \underset{N_y}{4}, \underset{N_z}{-3} \rangle$ $Q_1 = (\underset{P_x}{1}, \underset{P_y}{1}, \underset{P_z}{0})$

∴ plane ⟺ $-8(x-1) + 4(y-1) - 3(z) = 0$